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10EC046

M.Tech. Degree Examination, January 2011

Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Solve $\begin{bmatrix} 1 & 2 & 3 & 14 \\ 4 & 5 & 7 & 35 \\ 3 & 3 & 4 & 21 \end{bmatrix}$ by Gauss elimination method. **(05 Marks)**

b. Find the LU decomposition, with $\ell_{ii} = 1$, for a matrix,

$$A = \begin{bmatrix} 2 & 1 & 1 & -1 \\ 4 & 5 & 4 & -1 \\ -4 & 1 & 4 & 4 \\ 6 & -3 & 3 & 1 \end{bmatrix}.$$ **(07 Marks)**

c. Solve the system of equations

$x_1 + 2x_2 + x_3 = 0$

$2x_1 + 2x_2 + 3x_3 = 3$

$-x_1 - 3x_2 = 2$

by LU – factorization method with $u_{ii} = 1$. **(08 Marks)**

2 a. Prove that the non empty subset W of a vector space 'V' is a subspace of 'V' iff W is closed under vector addition and scalar multiplication. **(06 Marks)**

b. Show that the set B ={(1, 1, 0), (1, 0, 1) (0, 1, 1)} is a basis of the vector space $V_3(R)$. **(07 Marks)**

c. Find the dimension and basis of the subspace spanned by the vector (2 4 2) (1, -1 0) (1, 2, 1) and (0, 3, 1) in $V_3(R)$. **(07 Marks)**

3 a. Show that the sum of the two subspaces of the vector space V(F) is a subspace of V(F). **(06 Marks)**

b. $f: V_3(R) \to V_2(R)$ is defined by

$f(x, y, z) = (x - y, y + z)$. Show that 'f' is linear transformation. **(07 Marks)**

c. Find the co-ordinate vector of (3, -2, 1) relative to

i) Ordered standard basis

ii) The ordered basis {(1, 1, 1) (1, 0, 0) (1, 1, 0)} of R^3. **(07 Marks)**

4 a. Find the matrix of the linear transformation

$T: V_2(R) \to V_3(R)$ defined by

$T(x, y) = (x + y,\ x, 3x - y)$, with respect to

$B_1 = \{(1, 1)\ (3, 1)\}$

$B_2 = \{(1, 1, 1)\ (1, 1, 0)\ (1, 0, 0)\}$. **(06 Marks)**

b. State and prove rank and nullity theorem. **(10 Marks)**

c. Let 'T' be a linear operator on $V_3(R)$ defined by $T(a, b, c) = (3a, a-b, 2a + b + c)\ \forall$ a, b, c $\in V_3(R)$, prove that $(T^2 - I)(T - 3I) = 0$. **(04 Marks)**

5 a. Show that the linear map $T: V_3(R) \to V_3(R)$ defined by

$T(e_1) = e_1 + e_2$

$T(e_2) = e_2 + e_3$

$T(e_3) = e_1 + e_2 + e_3$

is non – singular and find its inverse. **(08 Marks)**

b. Let T be a linear operator on a vector space V. Let 'λ' be the eigen value of 'T'. Then show that:

i) K_λ is a T – invariant subspace of V containing E_λ.

ii) For any scalar $\mu \neq \lambda$, the restriction of $T - \mu I$ to K_λ is one – to – one. **(12 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Solve by orthogonally diagonalizing the matrix :

$$A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}.$$

(10 Marks)

b. Find the matrix p which diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, verify $\hat{P}AP = D$, where 'D' is the diagonal matrix and hence find A^6. (10 Marks)

7 a. Find the Q R factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. (10 Marks)

b. Find the least –square solution of AX = B for, $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$ by QR factorization.

(10 Marks)

8 Find the singular value decomposition of

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

(20 Marks)

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